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GLOBAL SOLUTIONS OF INITIAL VALUE PROBLEMS FOR SECOND ORDER NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS OF MIXED TYPE IN BANACH SPACES

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Abstract. In this paper, by use of Schauder's fixed point theorem, the existence of global solutions of initial value problem for second order integro-differential equation in a Banach space is investigated. As applications of the main theorem, the existence of global solutions of two classes of fourth order mixed boundary value problem are obtained.
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1 Introduction

Let $(E, \|\cdot\|)$ be a real Banach space, $J = [0, a],\ a > 0,\ x_0,\ x_1 \in E$. Now, we consider the following initial value problem (IVP) for nonlinear second order integro-differential equations of mixed type in E.

$$\begin{cases} x'' = f(t, x, x', Tx, Sx), & t \in J, \\ x(0) = x_0, \qquad x'(0) = x_1, \end{cases} \tag{1.1}$$

where $f \in C[J \times E \times E \times E \times E, E]$, the linear integral operator T and S are defined by

$$(Tx)(t) = \int_0^t k(t,s)x(s)ds, \quad (Sx)(t) = \int_0^a h(t,s)x(s)ds, \quad t \in J, \tag{1.2}$$

where $k \in C[D, R^1],\ h \in C[J \times J, R^1],\ D = \{(t,s) \in J \times J : s \le t\},\ R^1 = (-\infty, +\infty)$.

In the special case where f does not contain x' (see, (1.3)), the existence of maximal and minimal solutions of the following initial value problem for nonlinear second order integro-differential equation of mixed type in a Banach