

EXISTENCE RESULTS FOR EVOLUTION EQUATIONS VIA MONOTONE ITERATIVE TECHNIQUES

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Abstract. The aim of this paper is to find sufficient conditions for the existence of extremal solutions for discontinuous implicit initial value problems. We also prove the monotone dependence of the extremal solutions with respect to the data. An abstract monotone iterative technique and the theory of semigroups of operators are used.

Keywords. implicit evolution equations, extremal solutions, monotone iterative technique, semigroups of operators.

AMS(MOS) subject classification. 35K55, 34G20, 34A09, 47H15.

1 Introduction

The aim of this paper is to study the existence of extremal solutions for the time-dependent problems. We combine the monotone iterative technique (see [8,1,7,11,10,13,18]) and the theory of semigroups of operators (see [4,12,14,15]).

Let $X = (X, |\cdot|, \leq)$ be an ordered Banach space (OBS) with normal cone K , and let $T > 0$ be a given real number. Given $A : D(A) \subset X \rightarrow X$ a densely defined linear operator on X and a mapping $F : (0, T) \times X \rightarrow X$, we denote

$$Lu = u' + Au - F(t, u).$$

Given $u_0 \in X$ and a mapping $G : (0, T) \times X \times X \rightarrow X$, we consider the following initial value problem (IVP):

$$\begin{cases} Lu = G(t, u, Lu), & t \in (0, T) \\ u(0) = u_0. \end{cases} \quad (1)$$

We also consider the special case of IVP (1):

$$\begin{cases} u' + Au = G(t, u), & t \in (0, T) \\ u(0) = u_0. \end{cases} \quad (2)$$