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BOUNDARY OUTPUT SPECIFICATION IN CERTAIN DISTRIBUTED PARAMETER SYSTEMS

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Abstract. We examine, in the context of the Euler–Bernoulli and Rayleigh models for the elastic beam, the range of motion achievable at one end of the beam by means of control forces exerted at the other end. This non-traditional control problem is studied by combining the study of a certain "sideways", ill-posed Cauchy problem with modification of results long available on classical "state to state" controllability of elastic beams. Approximation and optimization possibilities are considered.
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1 Introduction

The movement of objects attached to one end of a robot arm, or space program manipulator arm, has been, and continues to be, a subject of wide engineering interest. Flexibility of the arm results in considerable difficulties in precise control of the remote end of the arm. Most studies directed toward resolution of this problem have emphasized various types of internal damping, or active control damping, of the arm, combined with study of the tracking properties of the remote end position output relative to the near end control input. There is little we can offer toward extension of these studies; rather, we will take an entirely different approach.

Our approach here is to study, in the context of two models of the arm, viewed as an elastic beam, the controllability of the remote end output with the near end input viewed as the control, asking for a more or less complete description of the achievable motions of the remote end for a given class of near end inputs. It will be seen that this is a distributed parameter control problem somewhat different from the "state to state" controllability problems classically studied for distributed parameter systems, as outlined, for example, in [8] and many other articles. The methods used are constructive and well adapted to computational approximation, optimization, etc. We will make some remarks in this direction at the end of the paper.

We know, of course, that it is possible to manipulate objects using flexible arms– "writing with a rubber pencil", as it were. If demonstration of that capability were our only, or even our main, objective here, our results would be meaningless. More realistically, work of the type we carry out here may