

ON OSCILLATION OF AN IMPULSIVE LOGISTIC EQUATION

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Abstract. For a scalar delay logistic equation

$$\dot{y}(t) = y(t) \sum_{k=1}^m r_k(t) \left(1 - \frac{y(h_k(t))}{K} \right), \quad h_k(t) \leq t,$$

under certain conditions such impulsive perturbations

$$y(\tau_j) - K = b_j (y(\tau_j - 0) - K), \quad \lim \tau_j = \infty,$$

can be introduced that the solution of the impulsive equation becomes oscillatory (eventually nonoscillatory). A connection between oscillation properties of the logistic delay impulsive equation and a linear impulsive equation

$$\begin{aligned} \dot{x}(t) + \kappa \sum_{k=1}^m r_k(t)x(h_k(t)) &= 0, \\ y(\tau_j) &= b_j (y(\tau_j - 0)), \quad \lim \tau_j = \infty, \end{aligned}$$

is investigated. Here κ is a constant close to one.

Keywords. Oscillation, nonoscillation, impulsive perturbations, logistic equation, impulsive control of oscillation properties.

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1 Introduction

The delay logistic equation

$$\dot{y}(t) = r(t)y(t) \left(1 - \frac{y(h(t))}{K} \right), \quad h(t) \leq t, \quad (1)$$

is known as Hutchinson's equation, if r and K are positive constants and the delay is constant $h(t) = t - \tau$, $\tau > 0$. Hutchinson's equation was investigated by several authors, see, for example, [13,14,17,21]. The oscillation of solutions of delay logistic equation (1) was investigated by Gopalsamy and Zhang [8,22]