

MULTIPLE SOLUTIONS OF SINGULAR IMPULSIVE BOUNDARY VALUE PROBLEM WITH POLYNOMIAL NONLINEARITY ¹

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Abstract. In this paper, we obtain some new results about the existence of multiple solutions for a singular impulsive boundary value problem with polynomial nonlinearity.

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1 INTRODUCTION

Consider the following impulsive singular boundary value problem

$$\begin{cases} y'' + \sum_{i=1}^m a_i(t)y^{\alpha_i} = 0, & 0 < t < 1, \quad t \neq t_1, \\ \Delta y|_{t=t_1} = I(y(t_1)), \\ y(0) = y(1) = 0. \end{cases} \quad (1.1)$$

with the following condition(see [1])

$$\Delta y'|_{t=t_1} = -\frac{I(y(t_1))}{1-t_1}. \quad (1.2)$$

where $\alpha_i < 0$, $1 \leq i \leq k$; $\alpha_i \geq 0$, $k+1 \leq i \leq m$, $\alpha_i \geq 1$, $k+r \leq i \leq m$, $m-1 \geq k > 0$, $r \geq 1$. $a_i(t) \in C((0,1), (0,+\infty))$, $R^+ = [0,+\infty)$, $I \in C(R^+, R^+)$, $\Delta y|_{t=t_1} = y(t_1^+) - y(t_1)$, $\Delta y'|_{t=t_1} = y'(t_1^+) - y'(t_1)$. Let $J = [0,1]$, $J_1 = [0,t_1]$, $J_2 = (t_1,1]$, $J' = J \setminus \{0,1,t_1\}$. Let $|J_i|(i=1,2)$ denote the length of interval $J_i(i=1,2)$.

Singular differential equations arise from many branches of mathematics and applied mathematics. It has been considered extensively, see [2]-[6] and their references. But most of these papers put their attentions to the existence of solutions for the singular differential equations and there were few papers to discuss the existence of multiple solutions for singular differential equations. Recently, in paper [6] Ravi P. Agarwal and O'Regan established some existence results of two solutions for a singular second order Dirichlet problem.

In this paper, we will establish the existence of two nonnegative solutions for the BVP (1.1) (1.2) by means of fixed point index theory. Different from

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