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ALMOST PERIODIC FUNCTIONS OF LYAPUNOV FOR IMPULSIVE DIFFERENTIAL EQUATIONS

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Abstract. Sufficient conditions for the existence of almost periodic piescewise continuous functions of Lyapunov's type for impulsive differential equations are obtained. The impulses take place at fixed moments of time.
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1. Introduction

Impulsive differential equations represent a natural apparatus for mathematical simulation of real processes and phenomena studied in phisycs, biology, population dynamics, biotechnologies, control theory, economics, etc. That is why in the recent years these equations have been the object of numerous investigations [1-5,7].

In this paper we shall prove convers theorem of the type of Massera's theorem [6], i.e. that for impulsive differential equations there exists a piecewise continuous almost periodic Lyapunov's function with certain properties.

2. Preliminary notes

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space with norm $||.||$, $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}^+ = [0, \infty)$, $D \subset \mathbb{R}^n$ is compact, $B_\alpha = \{x \in \mathbb{R}^n, ||x|| < \alpha\}$, $\alpha = const > 0$.

By B, $B = \{\{\tau_k\}_{k=-\infty}^{\infty} : \ \tau_k \in \mathbb{R}, \ \tau_k < \tau_{k+1}, \ k = 0, \pm 1, \pm 2, ...\}$ we denote the set of all sequences unbounded and strictly increasing with distance $\rho(\{\tau_k\}^{(1)}, \{\tau_k\}^{(2)}) = \inf_{\varepsilon > 0}\{|\tau_k^{(1)} - \tau_k^{(2)}| < \varepsilon, \ k = 0, \pm 1, \pm 2, ...\}$.

We shall consider the following system of impulsive differential equations

$$\begin{cases} \dot{x} = f(t, x), \ t \neq \tau_k, \\ \Delta x(\tau_k) = I_k(x(\tau_k)), \ k = 0, \pm 1, \pm 2, ..., \\ x(t_0 + 0) = x_0, t_0 \in \mathbb{R}, \end{cases} \tag{1}$$

where $f : \mathbb{R} \times D \to \mathbb{R}^n$, $I_k : D \to \mathbb{R}^n$, $\{\tau_k\}_{k=-\infty}^{\infty} \in B$, $\Delta x(\tau_k) = x(\tau_k + 0) - x(\tau_k)$.

We denote by $x(t) = x(t; t_0, x_0)$ the solution of (1) with the initial condition