

IMPULSIVE DIFFERENTIAL EQUATIONS WITH INITIAL TIME DIFFERENCE AND APPLICATIONS

Y. Zhang^{a,1} and B. Zhang^b

^aDepartment of Applied Mathematics
University of Waterloo,
Waterloo, Ontario, Canada N2L 3G1

^bDepartment of Statistics
Renmin University of China
Beijing, P.R.China, 100872

Abstract. In this paper, we study impulsive differential equations with initial time difference. Some inequalities and comparison theorem are established. Those results are used to study existence and stability properties of solutions.

AMS(MOS) subject classifications: 34D20, 34A37.

1 Introduction

It is well known that many phenomena in the real world can be described by impulsive differential equations and such equations have been studied widely^{[1]–[7]}. But, up to now, all the study of initial value problems of impulsive differential equations has been done only for perturbation or change of dependent variable keeping the initial time unchanged. However, to vary the initial time is very important as well since it is impossible to make no errors in the starting time. Recently, the investigation of initial value problems of differential equations with initial time difference has been initiated^{[8]–[11]}. Enlightened by these works, we initiate, in this paper, the investigation of impulsive differential equations with initial time difference and obtain some results in this new set up.

2 Notations

In this paper, we denote by $x \leq y$ if $x_i \leq y_i$ $i = 1, 2, \dots, n$, $|x| = (|x_1|, \dots, |x_n|)^T$, $\|x\| = \sum_{i=1}^n |x_i|$, where $x = (x_1, \dots, x_n)^T$, $y = (y_1, \dots, y_n)^T \in R^n$.

A function $f : R \times R^n \rightarrow R^n$ is said to be quasimonotone nondecreasing if $x \leq y$, $x_i = y_i$ implies $f_i(t, x) \leq f_i(t, y)$.

¹permanent address: Department of Mathematics, Xuzhou Normal University, Xuzhou Jiangsu, China, 221009.

The authors wish to express their sincere thanks to Professor V.Lakshmikantham for his help.