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BOUNDARY VALUE PROBLEMS FOR IMPULSIVE SECOND ORDER DIFFERENTIAL EQUATIONS

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Abstract. In this work, we investigate a problem of existence of solutions of the following system

$$x'' = f\left(t, x, x'\right), t \neq t_k$$

$$\begin{aligned} \Delta x\left(t_k\right) &= \eta_k\left(x\left(t_k\right)\right), \\ \Delta x'\left(t_k\right) &= \theta_k\left(x\left(t_k\right), x'\left(t_k\right)\right), k = 1, ..., r \end{aligned}$$

$$x(0) = x(1) = 0$$

We prove the existence of solutions using a nonlinear alternative.

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1. Introduction

Second order differential equations with impulsive effects arise naturally in the applied sciences to describe physical, biological or engineering processes that undergo abrupt changes at certain times. For a basic theory of impulsive differential equations we refer the reader to [2], [6]. Recently, several papers have been devoted to the study of second order impulsive differential equations (see for instance [4], [7], [9]).

In this paper we are concerned with the investigation of the existence of solutions of second order differential equations with impulsive effects. More specifically, we consider the following two-point boundary value problem for second order impulsive differential equations

$$x'' = f\left(t, x, x'\right), t \neq t_k \tag{1}$$

$$\Delta x\left(t_k\right) = \eta_k\left(x\left(t_k\right)\right), \tag{2}$$

$$\Delta x'\left(t_k\right) = \theta_k\left(x\left(t_k\right), x'\left(t_k\right)\right), k = 1, ..., r \tag{3}$$

$$x(0) = x(1) = 0 \tag{4}$$

Let $I' := I - \{t_k\}_{k=1}^{r}$. We suppose $f : I' \times R^2 \longrightarrow R$, is uniformly continuous, $\eta_k \in C(R; R), \theta_k \in C\left(R^2; R\right)$, $r \in N^*$. $\Delta x\left(t_k\right)$ denotes the jump of the function x at the point t_k, i.e. $\Delta x\left(t_k\right) = x\left(t_k^+\right) - x\left(t_k^-\right)$, where $x\left(t_k^+\right)$