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CONVERGENCE PROOF FOR RECURSIVE SOLUTION OF LINEAR-QUADRATIC NASH GAMES FOR QUASI-SINGULARLY PERTURBED SYSTEMS

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Abstract. A recursive method for the solution of the Riccati equations that yield the matrix gains required for the Nash strategies for quasi-singularly perturbed systems is presented. This method involves solution of Lyapunov and reduced-order Riccati equations corresponding to reduced-order fast and slow subsystems. The algorithm is shown to converge, under specified assumptions, to the exact solution with error at the kth iteration being $O(\epsilon^k)$ where ϵ is a small, positive, singular perturbation parameter.
Keywords. Nash equilibrium, matrix algebraic Riccati equation, quasi-singularly perturbed systems, recursive algorithm, differential games
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1 Introduction

The theory of linear-quadratic Nash games for singularly perturbed systems extends results from game theory, optimization, and the theory of singular perturbations. The decomposition of singularly perturbed systems into slow and fast modes, and consequences for the design and analysis of control systems, are presented in a collection of papers edited by Kokotovic and Khalil [9]. In game theory, Starr and Ho [16] derived the optimal Nash controls for nonzero sum differential games.

Nonzero-sum closed-loop Nash games with quadratic performance cost for singularly perturbed systems combine these two areas. Conditions for the well-posedness of these problems in the infinite time case were established by Gardner and Cruz [6] and by Khalil and Kokotovic [8]. Khalil [7] defined the concept of the near-equilibrium solution and showed that the approximate strategies obtained from the truncation of the Taylor series expansion