

## SINGULAR PERTURBATIONS AND TIME SCALES IN CONTROL THEORY AND APPLICATIONS: AN OVERVIEW<sup>1</sup>

D. Subbaram Naidu

Electrical Engineering and  
Measurement and Control Engineering Research Center  
College of Engineering, Idaho State University  
Pocatello, ID 83209-8060, USA

**Abstract.** This paper presents an overview of singular perturbations and time scales (SPaTS) in control theory and applications during the period 1984-2001 (the last such overviews were provided by [231, 373]). Due to the limitations on space, this is in way intended to be an exhaustive survey on the topic.

**Key Words:** Singular perturbations, time scales, control systems, order reduction, control theory and applications.

### 1 Introduction

A basic problem in the control system theory is the mathematical modeling of a physical system. The modeling of many systems calls for high-order dynamic equations. The presence of some “parasitic” parameters such as small time constants, resistances, inductances, capacitances, moments of inertia, and Reynolds number, is often the source for the increased order and “stiffness” of these systems. The stiffness, attributed to the simultaneous occurrence of “slow” and “fast” phenomena, gives rise to time scales. The systems in which the suppression of a small parameter is responsible for the degeneration (or reduction) of dimension (or order) of the system are labeled as “singularly perturbed” systems, which are a special representation of the general class of time-scale systems. The “curse” of dimensionality coupled with stiffness poses formidable computational complexities for the analysis and design of multiple time-scale systems.

From the perspective of systems and control, Kokotovic and Sannuti [374, 233, 375] were the first to explore the application of the theory of singular perturbations to continuous-time optimal control, both open-loop formulation leading to two-point boundary value problem [233] and closed-loop formulation leading to the matrix Riccati equation [375]. The methodology of

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