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Optimal Control of Linear Nonstandard Singularly Perturbed Discrete Systems

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Abstract. This paper introduces the definition of the nonstandard linear discrete-time singularly perturbed system and then shows how to solve the corresponding linear-quadratic optimal control problem since the methodology that exists in the literature for the solution of the standard singularly perturbed discrete linear-quadratic optimal control problem can not be extended to the corresponding nonstandard counterpart. The solution of the optimal control problem is obtained in terms of the pure-slow and pure-fast, reduced-order, *continuous-time*, algebraic Riccati equations. An example is included to illustrate the efficiency of the proposed method.
Keywords. Optimal control, Singular perturbation, Riccati equation, Order reduction, Parallel computation
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1 Introduction

Standard discrete-time linear singularly perturbed systems have been studied in the past by several researchers, see for example [3,4,5,6,12,13,15,16]. However, the nonstandard singularly perturbed *discrete* linear systems have not been yet defined in the control literature. Motivated by the existence of results for continuous-time nonstandard singularly perturbed linear systems [4,9,10,11,19], in this paper, we formally define the nonstandard singularly perturbed linear systems in discrete time, and show how to solve the corresponding linear-quadratic optimal control problem.

In this paper we use the formulation of singularly perturbed linear discrete-time control systems, based on the fast sampling model [13,14], given by

$$\begin{aligned} x_1(k+1) &= (I_{n_1}+\epsilon A_1)x_1(k)+\epsilon A_2x_2(k)+\epsilon B_1u(k), \quad x_1(0)=x_{10} \\ x_2(k+1) &= A_3x_1(k)+A_4x_2(k)+B_2u(k), \quad x_2(0)=x_{20} \end{aligned} \tag{1}$$

with slow variables $x_1 \in R^{n_1}$, fast state variables $x_2 \in R^{n_2}$, control inputs $u \in R^m$. ϵ is a small positive parameter. The system is said to be the standard form [13,14] when $(I_{n_2}-A_4)$ is nonsingular, otherwise it is said to be in the nonstandard form. Note that there are other models that have been developed for the discrete-time singularly perturbed systems. See [8,15,16] for the nonstandard assumptions for their models and for some more results.