Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **9 (2002)** 257-270

A NONLINEAR STRONG ERGODIC THEOREM FOR ASYMPTOTICALLY NONEXPANSIVE MAPPINGS WITH COMPACT DOMAINS

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Abstract. In this paper, we prove a nonlinear strong ergodic theorem for asymptotically nonexpansive mappings from a compact convex subset of a strictly convex Banach space into itself.
Keywords. Nonlinear ergodic theorem, fixed point, asymptotically nonexpansive mapping, convex approximation property.
AMS (MOS) subject classification: 47H09, 47H10.

1 Introduction

Throughout this paper, a Banach space is real and we denote by $\mathbf{N}$ and R^+, the set of all positive integers and the set of all nonnegative real numbers, respectively. Let C be a nonempty closed convex subset of a Banach space. A mapping $T : C \longrightarrow C$ is said to be asymptotically nonexpansive [6] if there exists a sequence $\{k_n\}$ of nonnegative real numbers with $\limsup_{n\to\infty} k_n \leq 1$ such that $\|T^n x - T^n y\| \leq k_n \|x-y\|$ for every $x, y \in C$ and $n \in \mathbf{N}$. T is said to be nonexpansive if $k_n = 1$ for all $n \in \mathbf{N}$. The first nonlinear ergodic theorem for nonexpansive mappings with bounded domains was proved by Baillon [2]: Let C be a nonempty bounded closed convex subset of a Hilbert space and let T be a nonexpansive mapping from C into itself. Then, for every $x \in C$, the Cesàro means $\frac{1}{n}\sum_{i=0}^{n-1} T^i x$ converge weakly to some $y \in F(T)$. Bruck [3] extended Baillon's theorem to a uniformly convex Banach space whose norm is Fréchet differentiable. Hirano and Takahashi [7] extended Baillon's theorem to an asymptotically nonexpansive mapping in Hilbert spaces. Oka [8] and Tan and Xu [10] extended Bruck's result to an asymptotically nonexpansive mapping in Banach spaces. On the other hand, Atsushiba and Takahashi [1] obtained the following nonlinear ergodic theorem for nonexpansive mappings with compact domains which generalizes Edelstein's result [5]: Let D be a nonempty closed convex subset of a strictly convex Banach space. Let T be a nonexpansive mapping from D into itself such that $T(D) \subset K$ for some compact subset K of D and let $x \in D$. Then, $\frac{1}{n}\sum_{i=0}^{n-1} T^{i+h} x$ converges strongly to a fixed point of T uniformly in $h \in \mathbf{N} \cup \{0\}$.