

Evolutionary Co-processes and Perturbations of Kato-stable Families of Operators

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Abstract. In this paper we introduce and study a new object, a *co-process* $U^\oplus(t, s)$, associated to an evolutionary process $U(t, s)$, $0 \leq s \leq t \leq T$, generated by a family $\{A(t)\}_{0 \leq t \leq T}$ of operators. Moreover, to prove a theorem of existence for the co-process, we obtain a theorem of Kato-stability for a family of operators with respect to perturbations $B(t) \in \mathcal{L}(D)$, where D is the constant domain of $A(t)$.¹

Key words: evolutionary process, non-autonomous linear abstract differential equation, co-process, family of generators of C_0 -semigroups, Kato-stability, perturbation.

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1 Introduction

Consider the problem

$$u'(t) = A(t)u(t), \quad (1)$$

where $\{A(t)\}_{0 \leq t \leq T}$ generates an evolutionary process $\{U(t, s)\}_{0 \leq s \leq t \leq T}$ on a Banach space X (see [7,10,11,12]).

In order to extend the variation-of-constants formula for multiplicative perturbation of (1) (see [5,6]), it can be useful to introduce a family of operators $U^\oplus(t, s)$ on X such that, for $0 \leq s \leq t \leq T$,

$$U^\oplus(t, s)|_{D(A(s))} = A(t)^{-1}U(t, s)A(s).$$

In Section 4 will be given more details on how to use the family $U^\oplus(t, s)$ to give a suitable definition of mild solutions.

Our aim is to find sufficient conditions to extend $U^\oplus(t, s)$ to X for all $t \geq s \geq 0$. Acquistapace and Terreni already treated the parabolic case in [1], Appendix, Prop. A1, ii).

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