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OSCILLATION CRITERIA FOR SECOND ORDER HALF–LINEAR DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS

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Abstract. Some new criteria for the oscillation of second order half–linear differential equations with deviating arguments of the form

$$\Big(a(t)|x'(t)|^{\alpha-1}x'(t)\Big)' - q(t)|x[g(t)]|^{\alpha-1}x[g(t)] \;=\; 0$$

are established.
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1. Introduction

In this paper we are concerned with the oscillatory behavior of half–linear functional differential equations of the type

$$\Big(a(t)|x'(t)|^{\alpha-1}x'(t)\Big)' - q(t)|x[g(t)]|^{\alpha}\ \mathrm{sgn}\ \ x[g(t)] \;=\; 0, \tag{1.1}$$

where

(i) α is a positive constant,

(ii) $q(t) \in C([t_0,\infty),[0,\infty)),\ q(t) \not\equiv 0$ eventually,

(iii) $g(t) \in C^1([t_0,\infty),\mathbb{R}),\ g'(t) \geq 0$ for $t \geq t_0$ and $\lim_{t\to\infty} g(t) = \infty$,

(iv) $a(t) \in C([t_0,\infty),(0,\infty))$, and

$$\int^{\infty} a^{-1/\alpha}(s)ds \;=\; \infty. \tag{1.2}$$

By a solution of equation (1.1), we mean a function $x \in C^1([T_x,\infty),\mathbb{R}),\ T_x \geq t_0$ which has the property that $a(t)|x'(t)|^{\alpha-1}x'(t) \in C^1([T_x,\infty),\mathbb{R})$ and satisfies equation (1.1) for all sufficiently large $t \geq T_x$. Our attention will be restricted to those solutions $x(t)$ of equation (1.1) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for all $T \geq T_x$. It is assumed that equation (1.1) does possess such a solution. A solution is said to be oscillatory if it