Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **9 (2002)** 201-215

The Algebraic Curve Solution for Riccati Equations with Polynomial Coefficients

Zhaosheng Feng

Department of Mathematics
Texas A&M University, College Station, TX 77843, USA
zsfeng@math.tamu.edu

Abstract. In this paper by the methods of algebraic curve and operator theory, we obtain the necessary and sufficient conditions for the existence of the algebraic curve solution for the Riccati equation with polynomial coefficients. By virtue of our results, for a given Riccati equation we can use the algorithm to judge whether or not it has the algebraic curve solution. If it does, we also can use the algorithm to work out its algebraic curve solution. The results are of significance in the qualitative theory of ordinary differential equations.
Keywords. Riccati equation, algebraic curve, existence, independence, algorithm.
AMS (MOS) subject classification: 30C15, 34A05, 34A34

1 Introduction

Consider the Riccati equation

$$y' = y^2 + P(x) \tag{1}$$

where $P(x)$ is a polynomial in $\mathbb{R}[x]$, i.e. $P(x) = \sum_0^K a_i x^i$ $(a_i \in \mathbb{R}, a_K \neq 0)$.

It is well-known that equation (1) plays an important role in the qualitative theory of ordinary differential equations, not only because many practical problems can be converted to equation (1), but also they can be widely applied in many scientific fields such as Engineering, Control Theory, Fluid Mechanics, and so on. Unfortunately, in the general case, equation (1) is not solvable[15], so numerical analysis is a common method by engineers and physicists. However, the problem that under what special conditions equation (1) is solvable has been a research topic for mathematicians during the past three decades (see the references [1-14, 16-19] and the references therein).

In this paper, by the methods of algebraic curve and operator theory, we obtain the necessary and sufficient conditions for the existence and uniqueness of the algebraic curve solution for equation (1). These results are not only very important in the qualitative theory of polynomial autonomous