Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **9 (2002)** 187-200

ASYMPTOTIC PERIODIC SOLUTIONS FOR A TWO-DIMENSIONAL LINEAR DELAY DIFFERENCE SYSTEM

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Abstract. In this paper we investigate asymptotic periodic behaviors of solutions to a linear delay difference system of dimension two

$$x_{n+1} - x_n + Ax_{n-k} = 0, \quad n = 0, 1, \cdots, \tag{L}$$

where k is a nonnegative integer and A is a 2×2 constant real matrix. We show that, when $A = p \begin{smallmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{smallmatrix}$ with $p = 2\cos\{(k\pi + |\theta|)/(2k+1)\}$, the critical value for the asymptotic stability, every solution of (L) is asymptotically periodic provided that θ/π is a rational number. And we also give explicit representations of such periodic solutions. Moreover, in the case A is given in other Jordan forms, asymptotic periodic behaviors of solutions are discussed.

AMS (MOS) subject classification: 39A10, 39A11

1. Introduction

Consider the linear delay difference system of dimension two

$$x_{n+1} - x_n + Ax_{n-k} = 0, \quad n \in \mathbb{Z}_+ = \{0, 1, \cdots\}, \tag{1}$$

where A denotes a 2×2 constant real matrix and k is a nonnegative integer.

Recently, Matsunaga and Hara [5] have obtained necessary and sufficient conditions for the asymptotic stability of (1) which improve the well known result, due to Levin and May [4] (see also [1; p.182], [2; p.12], [3], [6]), for the scalar difference equation

$$u_{n+1} - u_n + pu_{n-k} = 0, \quad n \in \mathbb{Z}_+. \tag{2}$$

Under the assumption that the matrix A is either of the Jordan forms

$$\text{(i)} \quad p \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \qquad \text{(ii)} \quad \begin{pmatrix} p_1 & q \\ 0 & p_2 \end{pmatrix},$$

they showed the following theorems, where p, θ, p_1, p_2 and q are all real constants and θ satisfies the condition $0 < |\theta| \le \pi/2$.

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