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# INTEGRABLE SOLUTIONS OF HAMMERSTEIN INTEGRAL INCLUSIONS IN BANACH SPACES

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**Abstract.** In this paper we present a common existence theory for both continuous and integrable solutions to Hammerstein inclusions in Banach spaces.
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# 1 Introduction

In this paper we present some very general existence theorems for the Hammerstein integral inclusion

$$u(t) \in \int_0^T k(t,s)\, g(s, u(s))\, ds \quad \text{a.e. } t \in [0,T]. \tag{1.1}$$

Here $k$ is a real single-valued function and $g$ is a set-valued map with values in a real Banach space $(E, |.|)$. In particular, if $k(t,s) = 0$ for $0 \le t < s \le T$ (the *Volterra case*), (1.1) becomes the Volterra integral inclusion

$$u(t) \in \int_0^t k(t,s)\, g(s, u(s))\, ds \quad \text{a.e. } t \in [0,T]. \tag{1.2}$$

Existence results for abstract Hammerstein integral equations and inclusions were obtained by several authors in the literature (see [1-3, 6, 7, 9, 10, 12, 13] and the references therein).

In our recent paper [11] we presented set-valued versions of Mönch's fixed point theorems and we applied them to the solvability of (1.1) and (1.2) in the space $C([0,T]; E)$. The aim of this paper is to show that the same technique can be used to discuss the solvability of (1.1) and (1.2) in $L^p([0,T]; E)$ ($1 \le p < \infty$). Moreover, we show that the two cases (of continuous solutions and of $L^p$-solutions ($1 \le p < \infty$), respectively) can be treated together by considering $1 \le p \le \infty$.

We conclude the introduction with some notations, preliminaries and well-known results which will be used in the next section.