Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **9 (2002)** 151-163

NONOSCILLATION RESULTS FOR NONLINEAR SYSTEMS WITH NONDECREASING ENERGY

John R. Graef[1], János Karsai[2], and Bo Yang[3]

[1]Department of Mathematics
University of Tennessee at Chattanooga
Chattanooga, TN 37403, USA
[2] Department of Medical Informatics
Albert Szent-Györgyi Medical University
Szeged, Korányi fasor 9, Hungary
[3] Department of Mathematics and Statistics
Mississippi State University
Mississippi State, MS 39762

Abstract. We consider the nonlinear system with impulsive perturbation

$$(\phi_\beta(x'))' + f(x) = 0 \quad (t \neq t_n), \quad x'(t_n + 0) = b_n x'(t_n)$$

where $n = 1, 2 \ldots$, $\phi_\beta(u) = |u|^\beta \text{sgn } u$ with $\beta > 0$, $uf(u) > 0$ for $u \neq 0$, and $b_n \geq 1$. We give criteria to guarantee that certain solutions of this system are nonoscillatory. We apply the results to the differential equation

$$(\phi_\beta(x'))' + q(t)f(x) = 0$$

with a nonincreasing step-function $q(t)$ and formulate sharp nonoscillation criteria.

AMS (MOS) subject classification: 34D05, 34D20, 34C15

1. Introduction

Consider the impulsively perturbed system

$$\begin{aligned} &(\phi_\beta(x'))' + f(x) = 0, \quad t \neq t_n, \\ &x(t_n + 0) = x(t_n), \quad x'(t_n + 0) = b_n x'(t_n), \end{aligned} \tag{1}$$

where $0 \leq t_1 < t_2, \ldots, t_n < t_{n+1}$, $t_n \to \infty$ as $n \to \infty$, $b_n \geq 1$ for $n = 1, 2 \ldots$, $\phi_\beta(u) = |u|^\beta \text{sgn } u$ with $\beta > 0$, $f : \mathbf{R} \to \mathbf{R}$ is continuous and odd, and $uf(u) > 0$ for u$\neq 0$. Define the energy function

$$V(x, y) = y\phi_\beta(y) - \int_0^y \phi_\beta(s)\, ds + \int_0^x f(s)\, ds =: \Phi_\beta(y) + F(x), \tag{2}$$

where $\Phi_\beta(y) = \frac{\beta}{\beta+1}|y|^{\beta+1}$. Note that the functions F and Φ_β are both even and positive definite.

It is easy to verify that $V(t) = V(x(t), x'(t))$ is constant along the solutions of the equation without impulses

$$(\phi_\beta(x'))' + f(x) = 0, \tag{3}$$