NONOSCILLATION RESULTS FOR NONLINEAR SYSTEMS WITH NONDECREASING ENERGY

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Abstract. We consider the nonlinear system with impulsive perturbation
\[
(\phi_\beta(x'))' + f(x) = 0 \quad (t \neq t_n), \quad x'(t_n + 0) = b_n x'(t_n)
\]
where \( n = 1, 2, \ldots \), \( \phi_\beta(u) = |u|^\beta \text{sgn } u \) with \( \beta > 0 \), \( uf(u) > 0 \) for \( u \neq 0 \), and \( b_n \geq 1 \).

We give criteria to guarantee that certain solutions of this system are nonoscillatory. We apply the results to the differential equation
\[
(\phi_\beta(x'))' + q(t)f(x) = 0
\]
with a nonincreasing step-function \( q(t) \) and formulate sharp nonoscillation criteria.

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1. Introduction

Consider the impulsively perturbed system
\[
(\phi_\beta(x'))' + f(x) = 0, \quad t \neq t_n, \quad x(t_n + 0) = x(t_n), \quad x'(t_n + 0) = b_n x'(t_n),
\]
where \( 0 \leq t_1 < t_2, \ldots, t_n < t_{n+1}, t_n \to \infty \) as \( n \to \infty \), \( b_n \geq 1 \) for \( n = 1, 2, \ldots \), \( \phi_\beta(u) = |u|^\beta \text{sgn } u \) with \( \beta > 0 \), \( f : \mathbb{R} \to \mathbb{R} \) is continuous and odd, and \( uf(u) > 0 \) for \( u \neq 0 \). Define the energy function
\[
V(x, y) = y\phi_\beta(y) - \int_0^y \phi_\beta(s) \, ds + \int_0^x f(s) \, ds =: \Phi_\beta(y) + F(x),
\]
where \( \Phi_\beta(y) = \frac{\beta}{\beta+1} |y|^\beta+1 \). Note that the functions \( F \) and \( \Phi_\beta \) are both even and positive definite.

It is easy to verify that \( V(t) = V(x(t), x'(t)) \) is constant along the solutions of the equation without impulses
\[
(\phi_\beta(x'))' + f(x) = 0,
\]