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LONG-TIME ERROR ESTIMATION AND A STABILITY-SMOOTHING INDICATOR

Tong Sun[1] and Richard E. Ewing[2]

[1]Department of Mathematics and Statistics
Bowling Green State University, Bowling Green, OH 43403
[2]Institute for Scientific Computation
Texas A&M University, College Station, Texas 77843

Abstract. An innovative approach of long-time error estimation for evolution equations is proposed in this paper. Instead of numerical error propagation, which is usually refered as the stability of a numerical scheme, we consider exact error propagation, which only depends on the stability of the dynamical system associated to the differential equation (DE) being solved. The tradeoff of using exact error propagation is that we need to establish the smoothness of the solutions of the DE which have the numerical solution as their initial values at each time step. To this end, we introduce a smoothing assumption on the DE and a stability-smoothing indicator on the numerical solution. The smoothing assumption should be verified as part of the mathematical study of the DE. The stability-smoothing indicator can be computed locally at each time step of the numerical solution. Combining the exact error propagation, a two level error propagation analysis technique, a new concept of moving attractors, the smoothing assumption and the stability-smoothing indicator, we can prove the general long-time error estimation theorems of this paper.
keywords. Long-time error estimation, stability-smoothing indicator, moving attractor, nonlinear dynamical system, numerical solution.
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1 Introduction

Long-time error estimation for numerical solutions of evolution equations is very important for both the theory of numerical analysis and the practice of scientific computation. To deal with long-time error, one has to carefully analyse error propagation. Traditionally, error propagation analyses in most textbooks are based on the stability concept on numerical schemes. But, for complex nonlinear systems treated by a combination of numerical techniques such as linearization, partially implicit schemes, local time-stepping, insufficient iterations of an implicit scheme in a local time step, etc., it is difficult and tedious, even if possible, to carry out error propagation analysis for numerical methods.

Numerical stability usually means the error propagation property of a scheme. For linear numerical schemes used to approximately solve linear evolution equations, because of the additivity, boundedness of solutions and