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OSCILLATION OF DELAY DIFFERENCE EQUATIONS: NEAR A CRITICAL CASE

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Abstract. This paper deals with oscillation problem for a linear delay difference equation with a variable coefficient. A general result is first established. From this general result, an explicit criterion is derived near a critical case. By comparing this criterion with some existing result on delay differential equations, an essential difference is identified between the linear delay difference equation and it's continuous version.
Keywords. Difference equation, eventually positive solution, oscillation

1 Introduction

In recent years there has been an increasing interest in oscillation problem of the delay difference equation (see [2]-[9])

$$x_{n+1} - x_n + p_n x_{n-k} = 0, \tag{1.1}$$

where $k > 0$ is an integer, which is the linearization of the delay Logistic difference equation

$$x_{n+1} = px_n \left(1 - \frac{x_{n-k}}{K}\right)$$

at its positive equilibrium $K\left(1 - \frac{1}{r}\right)$. Eq. (1.1) can also be obtained by discretizing the delay differential equation

$$x'(t) + p(t)x(t-r) = 0, \tag{1.2}$$

which is the linearization of the delay Logistic differential equation

$$x'(t) = p(t)x(t)\left(1 - \frac{x(t-r)}{K}\right)$$

at the positive equilibrium K, where $r \geq 0$ is a real constant.