Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **9 (2002)** 79-88

ON AN APPLICATION OF THE SADDLE-POINT THEOREM

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Abstract. We prove the existence of minimax type solution for equation $\Delta u + g(x,u) = h(x)$. We give conditions on both the nonlinearity and its potential.

AMS (MOS) subject classification: 1991 subject classification: 35J65.

1 Introduction and Summary

In this paper we apply Rabinowitz's Saddle-Point Theorem [14], to a semi-linear boundary value problem. We improve the results of a previous paper [16] published in September 98 issue of *Dynamic Systems and Applications*. Here we consider the elliptic boundary value problem

$$\begin{cases} \Delta u + g(x,u) = h(x) \\ u|\partial\Omega = 0 \end{cases} \tag{1}$$

$$(2)$$

where $\Omega \subset I\!R^N$, $N \geq 1$, is a bounded connected open set, and $h \in L^2(\Omega)$. As in [16], we leave aside questions of regularity and look for $H_0^1(\Omega)$-weak solutions of (1) and (2). Our main result is the following theorem.

Theorem *Let λ_k, λ_{k+1} be the k^{th} and $(k+1)^{\text{th}}$ eigenvalues of the problem $\Delta u + \lambda u = 0$, $u|\partial\Omega = 0$ respectively. Let $h \in L^2(\Omega)$ and g be continuous on $\Omega \times I\!R$. Let $G : \Omega \times I\!R \to I\!R$ be defined by*

$$G(x,\xi) = \int_0^{\xi} g(x,s)ds$$

Assume $\lambda_k < \lambda_{k+1}$ and