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EXISTENCE FOR A NONLINEAR ELLIPTIC SYSTEM AT RESONANCE

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Abstract. We will study the nonlinear elliptic system

$$-\triangle u = \lambda u + g(u) - v, \tag{0.1}$$

$$-\triangle v = \delta u - \gamma v, \tag{0.2}$$

subject to Dirichlet boundary conditions. Results on the existence of a nonzero solution for this system will be obtained by applying the Leray-Schauder degree theory. Application of the theorems will be shown by examples.

Keywords. Elliptic system, boundary value problem, eigenvalue, Laplacian o perator, degree theory

AMS (MOS) subject classification: 2000 Mathematics Subject Classification: 35J55, 35G30

1 Introduction

Let $\Omega \subset \mathbb{R}^N$ be a bounded smooth domain. The existence of nonzero solutions of the following elliptic system subject to Dirichlet boundary conditions $u = v = 0$ on $\partial\Omega$ was studied in [3], [4], [9]:

$$-\triangle u = \lambda u + g(u) - v, \tag{1.1}$$

$$-\triangle v = \delta u - \gamma v, \tag{1.2}$$

where λ is a parameter, g is a real valued function and δ, γ are two positive numbers.

It is known that the system (1.1), (1.2) represents a steady state case of reaction-diffusion systems of interest in Biology. Equation (1.2) can be solved for v in terms of u. Let $B : L^2(\Omega) \to H_0^1(\Omega)$ denote the solution operator under Dirichlet boundary conditions. Then problem (1.1), (1.2) becomes the following problem for u:

$$-\triangle u + Bu = \lambda u + g(u) \quad \text{in} \quad \Omega, \tag{1.3}$$

$$u = 0 \quad \text{on} \quad \partial\Omega. \tag{1.4}$$

The operator $B : L^2(\Omega) \to H_0^1(\Omega)$ is positive and selfadjoint and its spectrum is $\{\mu_k = \delta/(\gamma + \lambda_k), k = 1, 2, ...\}$, where λ_k denote the eigenvalues of the Laplacian with Dirichlet boundary conditions.