Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **9 (2002)** 35-42

BOUNDED SOLUTIONS OF A FORCED NONLINEAR INTEGRO-DIFFERENTIAL EQUATION

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Abstract. This paper studies the existence of bounded solutions of a forced nonlinear integro-differential equation. Some sufficient conditions for the existence of such solutions are obtained.
AMS (MOS) subject classifications: 34K10, 34K20

1 Introduction

Consider scalar, nonlinear, functional equations with finite delay distributed of the form

$$x'(t) + \int_0^r g(x(t), x(t-s))d\mu(s) = f(t) \tag{1}$$

where μ is a nondecreasing function in $[0, r]$, $g : \mathbb{R}^2 \to \mathbb{R}$ is at least continuously differentiable and the forcing function $f : \mathbb{R} \to \mathbb{R}$ is an arbitrary continuous function .

In this paper we study the existence of bounded solutions of (1). By a bounded solution of (1), we mean a function x that is defined, bounded in the whole real line and such that eq. (1) is satisfied for all $t \in \mathbb{R}$.

In order to motivate our results we review some previous studies. We first consider the linear equation,

$$x'(t) + ax(t) + bx(t-r) = f(t) \tag{2}$$

where $a > 0$, $b \in \mathbb{R}$, $r > 0$ and $f : \mathbb{R} \to \mathbb{R}$ is a continuous function. It is shown in the classical literature, (see [3]), that for $a > |b|$ equation (2) has a unique bounded solution for every bounded forcing function f.

A generalization of this result is given by Drager and Layton in [1] for equations related with (1). In [1], the authors suppose that μ is a finite positive Borel measure on the real line $\mathbb{R}$ and the delays are possibly infinite. The essential condition imposed to g in [1] is either

$$\begin{gathered} D_1 g(x,y) > |D_2 g(x,y)| \quad \forall (x,y) \in \mathbb{R}^2 \\ \text{or} \\ -D_1 g(x,y) > |D_2 g(x,y)| \quad \forall (x,y) \in \mathbb{R}^2. \end{gathered} \tag{3}$$