*Dynamics of Continuous, Discrete and Impulsive Systems*
*Series* **A:** *Mathematical Analysis* **9 (2002)** 19-28


# STABLE SET OF ABSTRACT NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS*

Shuyong Li[1,2†], Daoyi Xu[1‡] and Xiaoping Zhou[2]
[1]Department of Mathematics
Sichuan University
Chengdu 610064, P. R. China
[2]Department of Mathematics
Sichuan Normal University,
Chengdu 610066, P. R. China

**Abstract.** This paper is concerned with asymptotically or exponentially stable set of the equilibrium 0 of a class of abstract nonlinear functional differential equations. Sufficient conditions for determining the stable set are given by the semigroup theory, the inequality techniques and the properties of nonnegative matrices. The main theorems are illustrated by two examples.
**AMS(MOS) subject classification:** 34G20, 34K20

## 1. Introduction

In the last a few years, the stability and asymptotic behavior of (functional) differential equations have been extensively discussed (see, [1], [3], [4], [5], [7], [11], [12], [13], [14], [15]) and various results have been reported. These results are valid either globally in the entire state space or locally in the neighborhood of the equilibrium states. However both types of results are unsatisfactory in a certain sence. A large class of models, such as equation $\frac{dx(t)}{dt} = -ax(t) + bx(t)x(t-\tau)$, have multiple equilibrium which exclude globally asymptotic stability. On the other hand, local results are unsatisfactory in that it is not certain how far can initial conditions be allowed to vary without disrupting the stability properties established in the immediate vicinity of equilibrium states? These ideas lead to the problem of investigating the stable set of differential equations. In [1], stable set of the equilibrium 0 of functional differential equations was discussed. In [7], stable set of the equilibrium 0 of partial functional differential equations was obtained. In [11], stability region (*i.e.* stable set) of the equilibrium 0 of large-scale dynamic systems was estimated. In [15], K-stability, a class of stable set, of nonlinear systems was studied.

The purpose of the paper is to derive easy check conditions for the stable set in the norm of the equilibrium 0 of abstract nonlinear functional differ-

*Part of this research was supported by NNSF of China and by YFP of Sichuan of Sichuan Province Education Commission
†The author now is working in Sichuan Normal University
‡Corresponding author, E-mail: dyxu@scuu.edu.cn