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BOUND SETS FOR FIRST ORDER DIFFERENTIAL EQUATIONS WITH GENERAL LINEAR TWO-POINTS BOUNDARY CONDITIONS

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Abstract. We consider differential equations with linear two-points boundary conditions. We present some existence results for bound sets defined as the intersection of sublevel sets of particular scalar functions, called bounding functions. All the three cases, namely continuous, locally lipschitzian and C^1-class bounding functions, are analized. Comparisons with previous results are given. Finally we apply the existence theorems to the homogeneous Cauchy problem and to the Picard problem.
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1 Introduction

In [5] (see Theorems 1.3 and 2.3 and Remark 2.5), using a continuation theorem for Fredholm maps of index zero (we remind to [1] and [4] for a theory on this subject), we proved an existence theorem for the solution of the boundary value problem

$$\begin{cases} x' = f(t,x), & t \in [a,b] \\ Ax(a) + Bx(b) = 0, \end{cases} \tag{1}$$

where $f : [a,b] \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function and A and B are $n \times n$ real valued matrices. Recalling the notation used in that paper, i.e. P is the continuous projection of $\mathbb{R}^n$ onto $\mathrm{Im}(A+B) \cap \mathrm{Im}B$ and d is the topological degree, the theorem is the following:

Theorem 1 *If $dimIm(A+B) + dimImB - dim(Im(A+B) \cap ImB) = n$ and there exists $G \subset \mathbb{R}^n$ such that*

1) $\forall\, \lambda \in (0,1)$ G is a bound set for

$$\begin{cases} x' = \lambda f(t,x), & t \in [a,b] \\ Ax(a) + Bx(b) = 0, \end{cases}$$