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THE INVARIANT CURVE CAUSED BY NEIMARK-SACKER BIFURCATION

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Dedicated to Professor Yoshinori Kametaka on his 60th birthday

Abstract. We derive the formula to compute the stability conditions of the invariant curve caused by the Neimark-Sacker bifurcation. Moreover, we give the explicit expression of the invariant curve. We apply our result to the delayed logistic equation.
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1 Introduction

Consider a difference equation depending on one parameter

$$x_{n+1} = f_\mu(x_n), \quad x_n \in \mathbb{R}^m, \quad \mu \in \mathbb{R} \tag{1.1}$$

where f_μ is a smooth function, and μ is a parameter. Suppose that (1.1) has a fixed point x^* for μ sufficiently small. Transforming the fixed point x^* to the origin, we can write (1.1) as

$$x_{n+1} = Ax_n + F_\mu(x_n), \quad x_n \in \mathbb{R}^m, \quad \mu \in \mathbb{R} \tag{1.2}$$

where A is the Jacobian matrix of f_μ evaluated at the fixed point x^*, and $F_\mu(x) = f_\mu(x^* + x) - f_\mu(x^*) - Ax$. In the following, we assume that A has a pair of simple eigenvalues $\lambda(\mu)$, $\bar{\lambda}(\mu)$ satisfying

(A1) $\quad |\lambda(0)| = 1$
(A2) $\quad \lambda(0)^j \neq 1$ for $j = 1, 2, 3, 4$
(A3) $\quad \left.\dfrac{d}{d\mu}|\lambda(\mu)|\right|_{\mu=0} \neq 0$

and the other eigenvalues have moduli less than 1 for μ sufficiently small. Under some additional conditions on the nonlinear term F_μ, the invariant curve can be created around the fixed point x^* when the parameter μ is varied near zero. This phenomenon is called *a Neimark-Sacker bifurcation* ([1, 2, 5]).