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ON A CONFLUENT HYPERGEOMETRIC FUNCTION OF TWO VARIABLES

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Abstract. R. Garnier derived a system of partial differential equations called an N-dimensional Garnier system, which is a generalization of the sixth Painlevé equation P_{VI} . The 2-dimensional Garnier system G_2 admits particular solutions expressed in terms of the Appell's hypergeometric function $F_1(\alpha,\beta,\beta',\gamma,x,y)$ and degenerate systems derived from G_2 also admit ones expressed in terms of confluent hypergeometric functions of two variables obtained from $F_1(\alpha,\beta,\beta',\gamma,x,y)$. In this paper we treat one of them, give a triple of linearly independent solutions of this system expanded into convergent series, and near the irregular singular locus $x=\infty$, examine the asymptotic behaviour of solutions.
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1 Introduction

Let z_C be a function of $(x,y)\in \mathbf{C}^2$ defined by

$$z_C(x,y)=\int_C \exp\left(\frac{t^3}{3}+yt^2+xt\right)t^{-\alpha-1}dt. \tag{1}$$

Here $\alpha(\notin \mathbf{Z})$ is a complex constant and C is a path such that the integrand vanishes at its terminal points. It is known that $z_C(x,y)$ satisfies a system of partial differential equations of the form

$$\begin{aligned}{\partial_x}^2u &= \partial_y u,\\ \partial_x\partial_y u &= -x\partial_x u-2y\partial_y u+\alpha u,\\ {\partial_y}^2 u &= (2xy+\alpha-1)\partial_x u+(4y^2-x)\partial_y u-2\alpha y u,\end{aligned} \tag{2}$$

whose solutions constitute a 3-dimensional vector space over $\mathbf{C}$ (cf.[4]). This system is equivalent to

$$dV=(P(x,y)dx+Q(x,y)dy)V \tag{3}$$

with

$$\begin{aligned} P(x,y) &= \begin{pmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ \alpha & -x & -2y\end{pmatrix}, \quad Q(x,y)=\begin{pmatrix} 0 & 0 & 1\\ \alpha & -x & -2y\\ -2\alpha y & 2xy+\alpha-1 & 4y^2-x\end{pmatrix},\\ V &= {}^t\left(u,\partial_x u,\partial_y u\right).\end{aligned}$$