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MELNIKOV ANALYSIS FOR A DISSIPATIVE FIFTH-ORDER SYSTEM

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Abstract. The normal form theory is applied to a dissipative fifth-order system of magnetoconvection in order to explain its numerical solutions qualitatively. The normal form equation reveals that the two typical phase portraits numerically obtained can be explained by the Duffing equation and the van der Pol equation. The Melnikov function for the perturbed system is obtained from the normal form equation. It is shown that an arbitrary parameter contained in the normal form equation can be determined consistently in terms of the relation between the nonlinear stability and the Melnikov analysis.
Keywords. Melnikov analysis, dissipative system, normal form theory, nonlinear stability

1 Introduction

Many physical systems are subjected to purely temporal dynamics and are characterized by strong dissipation so that the long-term behavior can be described by low-dimensional attractors [1] in ordinary differential equations(ODE) instead of the partial differential equations(PDE). Lorenz [2] found the concept of the strange attractor from his truncated model, which is a third-order system of ordinary differential equations instead of the partial differential equations designed to represent the Rayleigh-Bénard convection in fluids. Franceshini and Tebaldi [3] studied two infinite sequences of orbits leading to turbulence in a five-mode truncation of the Navier-Stokes equation for a two-dimensional incompressible fluid. Knobloch and co-workers [4, 5, 6, 7] also introduced two slightly different fifth-order systems of equations that reduce to the Lorenz system when one of the parameters is set equal to zero. One set of equations is derived from a thermosolutal convection problem, the other from the problem of two-dimensional convection interacting with magnetic fields, which is called magnetoconvection [8]. Nonlinear interaction between convection and magnetic fields may explain certain prominent features on the solar surface. Magnetic fields within sunspots are sufficiently strong to suppress convection on granular and supergranular scales [9]. Magnetoconvection exhibits a rich variety of behavior when the magnetic Prandtl number (the ratio of the magnetic to the thermal diffusivity) is small. The fifth-order system of magnetoconvection is a straightforward extension of the Lorenz model for the Boussinesq convection interacting