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PRECISE COMPUTATION OF HOPF BIFURCATION AND TWO APPLICATIONS

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Abstract. An algorithm is described for the computation of Hopf bifurcation coefficients for autonomous ordinary differential systems, by Lyapunov-Schmidt reduction. Interval values for the coefficients are found and it is known with mathematical certainty that the exact values fall in the computed intervals. The algorithm presented is a precise version of a scheme developed by Shengli Wang (PhD dissertation, SUNY Buffalo 1994), to solve the location and recognition problems when the (singularity theoretic) normal form for the bifurcation has $codim_{\mathbb{Z}_2} \leq 3$. For certain normal forms, the methods also allow solution of the universal unfolding problem. Applications to an enzyme-catalyzed reaction model and to the Hodgkin-Huxley nerve conduction model are given.

Keywords. Hopf bifurcation, Lyapunov-Schmidt reduction, recognition problem, universal unfolding, interval analysis.

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1 Introduction

Consider an autonomous ordinary differential system of the form:

$$\frac{dX}{dt} = f(X, \nu^1, ..., \nu^m) \tag{1}$$

where X, f are in $\mathbb{R}^n, (\nu^1, ..., \nu^m)$ in $\mathbb{R}^m$ and f is smooth, $f = (f_1, ..., f_n)$. Then a Hopf bifurcation point $(X_b, \nu_b^1, ..., \nu_b^m)$ in $\mathbb{R}^{n+m}$ is a point such that i)$f(X_b, \nu_b^1, ..., \nu_b^m) = 0$, ii) the Jacobian matrix $A_b = D_X f(X_b, \nu_b^1, ..., \nu_b^m)$ has a single pure imaginary pair of eigenvalues, and iii) all other eigenvalues of A_b have negative real parts.

For $(\nu^1, ..., \nu^m)$ near $(\nu_b^1, ..., \nu_b^m)$ the non-trivial periodic solutions of (1) which are close to X_b correspond to small positive solutions u of a bifurcation equation $r(u, \nu^1 - \nu_b^1, ..., \nu^m - \nu_b^m) = 0$, where the function r is here obtained by Lyapunov-Schmidt reduction. We chose Lyapunov-Schmidt reduction because it is widely understood. There are other methods: Farr, Li,