

## BIFURCATION AND NORMAL FORM OF PARAMETRICALLY EXCITED DUFFING SYSTEM WITH QUADRATIC DAMPING

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**Abstract.** In this paper, we analyze the bifurcation of parametrically excited Duffing system with quadratic damping. Modulation equations expressing the time variation of the amplitude and phase are derived by the method of multiple scales under the aide of symbolic manipulation. The normal forms on the center manifold are shown in the vicinity of the bifurcation points. Massive calculation in the derivation of the normal forms is avoided by estimating the order of the deformation of the center subspace due to the nonlinear effect in advance, which makes it unnecessary to express the center manifold explicitly. From obtained normal form, the changes of the number and the nonlinear characteristics of the bifurcation due to the magnitude of the quadratic damping are analytically clarified.

**Keywords.** Parametric Resonance, Duffing system, Method of Multiple Scales, Center Manifold, Normal Form, and Bifurcation.

## 1 Introduction

Dynamics of parametrically excited system is governed by Mathieu' Equation as:

$$\ddot{x} + (1 + \epsilon \sin \nu t)x = 0. \quad (1)$$

The system is attracted much attention by many researchers in some decades (the detail review is seen in the article [8]), because it corresponds to models of many mechanical systems such as surface waves in a fluid-filled cylinder under vertical excitation (for example [3, 5, 10]), the mechanical system supported by unsymmetrical restoring force [9, 13], and slender structure under axial periodic load [2], and so on. It is well known for linear parametrically excited system as Eq. (1) that nontrivial steady state is generally indefinite and hence the magnitude of the limit cycle cannot be determined from only linear term. From such a reason, by including cubic nonlinear term the limit cycle and other complex phenomena have widely been analyzed for