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PETROV-GALERKIN METHODS FOR NONLINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. This paper presents a class of Petrov-Galerkin finite element (PGFE) methods for the initial-value problem for nonlinear Volterra integro-differential equations:

$$y'(t) = f(t, y(t)) + \int_0^t k(t, s, y(s))ds, \quad t \in I := [0, T], \quad y(0) = 0.$$

These methods have global optimal convergence rates, and have certain global and local super-convergence features. Several post-processing techniques are proposed to obtain globally super-convergent approximations. As by products, these super-convergent approximations can be used as efficient a-posteriori error estimators. Numerical examples are provided to illustrate properties of these methods.

Keywords. Volterra integro-differential equations, Petrov-Galerkin finite element methods, optimal error estimates, interpolation post-processing, a-posteriori error estimators.
AMS (MOS) subject classification: 65R20, 65B05, 65N30.

1 Introduction

In this paper we discuss a class of Petrov-Galerkin finite element (PGFE) methods for the initial-value problem of nonlinear Volterra integro-differential equations: Find $y = y(t)$ such that

$$y'(t) = f(t, y(t)) + \int_0^t k(t, s, y(s))ds, \quad t \in I := [0, T], \quad y(0) = 0, \qquad (1.1)$$

where $f = f(t, y) : I \times R \to R$ and $k = k(t, s, y) : D \times R \to R$ (with $D := \{(t, s) : 0 \le s \le t \le T\}$) denote given functions.

It will always be assumed that the problem (1.1) possesses a unique solution $y \in C^1(I)$, which is guaranteed when the given functions $f(t, y)$ and $k(t, s, y)$ are, respectively, continuous for $t \in I$ and $(t, s) \in D$, with the the following (uniform) Lipschitz conditions [6]:

$$(V1) \quad |f(t, y_1) - f(t, y_2)| \le L_1 |y_1 - y_2|,$$
$$(V2) \quad |k(t, s, y_1) - k(t, s, y_2)| \le L_2 |y_1 - y_2|,$$