Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **8 (2001)** 315-338

ROBUST CONTROL OF THE KURAMOTO-SIVASHINSKY EQUATION

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Abstract. Robust control theory, a generalization of optimal control theory, has been proposed as an effective technique when control algorithms are sensitive to a broad class of external disturbances. In [6], a general framework for the robust control of the Navier-Stokes equations in finite time horizon was developed. In this article the robust boundary control for the Kuramoto-Sivashinsky equation is considered in the same spirit: a robust boundary control problem is formulated, and the existence and uniqueness for the robust control problem are proved. A data assimilation problem corresponding to the Kuramoto-Sivashinsky equation is considered, existence and uniqueness of solution are derived. This approach is also applicable as well to other equations with a structure similar to that of the Kuramoto-Sivashinsky equation.
Keywords. Robust control, Kuramoto-Sivashinsky equation, Energy estimate, Saddle points, Data assimulation problem.
AMS (MOS) subject classification: 49J20, 35Qxx, 34K35.

1 Introduction

In this article we address the problem of robust boundary control problem of the Kuramoto-Sivashinsky equation (KS equation for brevity) in its non-dimensional form:

$$u_t + u_{xxxx} + \lambda u_{xx} + uu_x = 0, \tag{1}$$

where $\lambda > 0$ is the "anti-diffusion" parameter. The KS equation has been introduced by Kuramoto [17] in space dimension 1 for the study of phase turbulence in the Belousov-Zhabotinsky reactions. An extension of this equation to space 2 (or more) has been introduced by Sivashinsky [22] in studying the propagation of a flame in the case of mild combustion. Since its appearance this equation has been intensively studied by many authors from the points of view of well-posedness and dynamics; we mention in particular [19, 20], who proved the long time stability for the odd solutions of (1) satisfying the periodic boundary condition and the existence of the global attractor, and [11] who established the existence of inertial manifolds; on the other hand,