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SOME PROPERTIES OF MONOTONE GRADIENT SYSTEMS

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Abstract. We prove maximal dissipativity of some gradient systems having a convex potential.
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1 Introduction

We are here concerned with the following operator in a separable Hilbert space H.

$$N_0\varphi = \frac{1}{2}\,\mathrm{Tr}\,[D^2\varphi] - \langle x, AD\varphi\rangle - \langle DU(x), D\varphi\rangle, \tag{1.1}$$

where $A : D(A) \subset H \to H$ is a self–adjoint negative operator, and $U : \mathbb{R} \to (-\infty, +\infty]$ is a convex nonnegative lower semicontinuous proper function ([1]).

Our goal is to show that N_0, with a properly defined domain, is essentially self–adjoint in $L^2(H, \nu)$ where ν is the measure $\nu(dx) = \rho(x)\mu(dx)$:

$$\rho(x) = Z^{-1}e^{-2U(x)},\ x \in H,\ Z = \int_H e^{-2U(y)}\mu(dy),$$

and μ is the gaussian measure with mean 0 and covariance operator $Q = -\frac{1}{2}\,A^{-1}$.

Let us give precise assumptions.

Hypothesis 1.1 *(i) $A : D(A) \subset H \to H$ is self–adjoint, and there exists $\omega > 0$ such that $A \le -\omega$. Moreover $Q := -\frac{1}{2}\,A^{-1}$ is of trace class.*

(ii) $U : H \to [0, +\infty]$ is convex, lower semicontinuous and non identically equal to $+\infty$.

[1] that is not identically equal to $+\infty$