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POSITIVE SOLUTIONS OF SINGULAR STURM-LIOUVILLE BOUNDARY VALUE PROBLEMS

B.G. Zhang and Lingju Kong

Department of Mathematics
Ocean University of Qingdao
Qingdao 266003 P.R. China

Abstract. This paper studies the Sturm-Liouville boundary value problem

$$\begin{cases} (p(t)u'(t))' + \lambda a(t)f(t,u(t)) = 0, \ 0 < t < 1, \\ \alpha u(0) - \beta p(0)u'(0) = 0, \\ \gamma u(1) + \delta p(1)u'(1) = 0, \end{cases}$$

where $\lambda > 0$ and a is allowed to be singular at both end points $t = 0$ and $t = 1$. We shall show the existence of this problem for λ on a suitable interval.

Keywords. Positive Solutions, boundary value problems, singular, fixed point theorem, cone.
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1 Introduction

Consider the Sturm-Liouville boundary value problem

$$\begin{cases} (p(t)u'(t))' + \lambda a(t)f(t,u(t)) = 0, \ 0 < t < 1, \\ \alpha u(0) - \beta p(0)u'(0) = 0, \\ \gamma u(1) + \delta p(1)u'(1) = 0, \end{cases} \tag{1$_\lambda$}$$

where
(H_1) $p(t) \in C([0,1],[0,+\infty))$ and $0 < \int_0^1 \frac{dt}{p(t)} < +\infty$;
(H_2) $\lambda > 0$, α, β, γ and δ are nonnegative, and $\beta\gamma + \alpha\gamma + \alpha\delta > 0$;
(H_3) $f(t,u) \in C([0,1] \times [0,+\infty), R^+)$ and $a \in C((0,1),[0,+\infty))$;
(H_4) $0 < \int_0^1 G(s,s)a(s)ds < +\infty$,
where

$$G(s,s) = \frac{1}{\rho}(\beta + \alpha \int_0^s \frac{dr}{p(r)})(\delta + \gamma \int_s^1 \frac{dr}{p(r)}), \ 0 \le s \le 1,$$

and

$$\rho = \alpha\delta + \alpha\gamma \int_0^1 \frac{dr}{p(r)} + \beta\gamma.$$

For any $t \in [0,1]$, let

$$f_0(t) = \lim_{u \to 0^+} \frac{f(t,u)}{u} \text{ and } f_\infty(t) = \lim_{u \to +\infty} \frac{f(t,u)}{u}.$$