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STABILITY OF NONLINEAR DIFFERENCE INCLUSIONS

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Abstract. The stability of difference inclusions $x_{k+1} \in \mathbb{F}(x_k)$ is studied, where $\mathbb{F}(x) = \{F(x,\lambda) : \lambda \in \Lambda\}$ and the selections $F(\cdot,\lambda) : E \to E$ assume values in a Banach space E, partially ordered by a cone K. It is assumed that the operators $F(\cdot,\lambda)$ are heterotone or pseudoconcave. The main results concern asymptotically stable absorbing sets, and include the case of a single equilibrium point. The results are applied to a number of practical problems.

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1. Introduction

A precise description of processes occurring in sampled-data systems is often unknown. A general model which incorporates such uncertainties in discrete-time systems is the difference inclusion

$$x_{k+1} \in \mathbb{F}(x_k), \tag{1}$$

where $\mathbb{F}$ is a set-valued mapping [1, 2, 7]. Commonly, $\mathbb{F}$ is described by a set of selections $F(\cdot,\lambda)$,

$$\mathbb{F}(\cdot) = \{F(\cdot,\lambda) : \lambda \in \Lambda\}. \tag{2}$$

Thus every trajectory of (1) is defined by

$$x_{k+1} = F(x_k,\lambda_k), \quad \lambda_k \in \Lambda, \tag{3}$$

for some sequence $\{\lambda_k\} \subset \Lambda$.

Note that the representation (2) does not imply that only parametric uncertainty is present and there may also be structural uncertainty. Clearly, different families of selections might represent the same setvalued mapping $\mathbb{F}$. See, for example the Castaing Representation Theorem [5].

The stability of such systems has been widely studied: see [3, 4, 6, 7, 8, 9, 10] and references therein for a representative selection. The principal thrust of the results below is the exploitation of properties phrased in terms of partial ordering in the state space. Definitions and notation are in the next section and the main results on asymptotic stability of fixed points and