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PERIODIC SOLUTIONS OF SINGULARLY PERTURBED NONLINEAR DIFFERENTIAL EQUATIONS WITH SYMMETRIES

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Abstract. This paper is devoted to the study of symmetric periodic solutions of the second order scalar differential equation

$$\frac{d^2x}{d\tau^2} + f(\varepsilon\tau, x) = 0$$

where ε is a positive and small real parameter and f is a function 2π-periodic in the first variable and satisfies some symmetry properties. After a description of the geometry of the phase diagram, the existence of different kinds of periodic solutions is proved, some of them being of oscillatory type at any time, otherones having very big amplitude at sometimes. These results generalize and extend the ones obtained in collaboration with B.V. SCHMITT in [17] for the equation of the slowly and periodically forced pendulum.
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1 Introduction

We consider a slowly and periodically oscillating system described by the differential equation

$$\frac{d^2x}{d\tau^2} + f(\varepsilon\tau, x) = 0 \tag{1}$$

where ε is a positive and small real number and f is a C^1 function 2π-periodic in the first variable. After the change of time $t = \varepsilon\tau$, equation (1) becomes

$$\varepsilon^2 \frac{d^2x}{dt^2} + f(t, x) = 0 \tag{2}$$

which is a $\frac{2\pi}{\varepsilon}$-periodic equation of the singularly perturbation type. Examples of (2) are the equation of the parametric pendulum

$$\varepsilon^2 \frac{d^2x}{dt^2} + (\alpha + \beta \cos 2t) \sin x = 0 \tag{3}$$