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On the Structure of the Set of Stationary Solutions for a Lotka-Volterra Competition Model with Diffusion

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Dedicated to Professors Masayasu Mimura and Takaaki Nishida on their sixtieth birthday

Abstract. In this paper, we study the structure of the set of stationary solutions for a Lotka-Volterra competition model with diffusion. To do this, the comparison principle and the bifurcation theory are employed.
Keywords. Lotka-Volterra model, comparison principle, bifurcation theory.
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1 Introduction

This paper is concerned with stationary solutions of a Lotka-Volterra competition model with diffusion

$$\begin{cases} \mathbf{u}_t = \varepsilon\, D\, \mathbf{u}_{xx} + \mathbf{f}(\mathbf{u}), \quad x \in (0,1), \quad t > 0, \\ \mathbf{u}_x = \mathbf{0}, \quad x = 0, 1, \quad t > 0 \end{cases} \tag{1.1}$$

with suitable initial condition, where $\mathbf{u} = (u, v)$, $D = \mathrm{diag}(1, d)$, $\mathbf{f} = (f, g)$,

$$f(\mathbf{u}) = (1 - u - c\,v)\,u, \quad g(\mathbf{u}) = (a - b\,u - v)\,v,$$

and every parameter is a positive constant. As u and v mean the population density, we restrict our discussion to positive solutions of (1.1), where we say that $\mathbf{u}$ is *positive* if $\mathbf{u}$ is in the first quadrant. It is easy to check that $\mathbf{f}(\mathbf{u}) = \mathbf{0}$ with $u \geq 0$ and $v \geq 0$ has the solutions $\mathbf{e}_0 = \mathbf{0}$, $\mathbf{e}_1 = (0, a)$, $\mathbf{e}_2 = (1, 0)$, and

$$\mathbf{e}_3 = \left(\frac{a - b}{1 - b\,c}, \frac{1 - a\,c}{1 - b\,c} \right)$$

which exists for either $b < a < 1/c$ or $1/c < a < b$.

First of all, we consider the case where $b < a$ and/or $a < 1/c$ are satisfied. Let $\mathbf{u}(x) = (u, v)(x)$ be an arbitrary stationary solution of (1.1) satisfying