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THE SIMPLEST NORMAL FORM FOR THE SINGULARITY OF A PURE IMAGINARY PAIR AND A ZERO EIGENVALUE

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Abstract. This paper is concerned with further reduction of the conventional normal forms of differential equations. The attention is focused on the case when the Jacobian of a system evaluated at an equilibrium has a pair of purely imaginary eigenvalues and a simple zero. Explicit, recursive formulas are derived, which can be used to compute the coefficients of the simplest normal form and the associated nonlinear transformation *up to any order*. It is shown that unlike conventional normal forms, the simplest normal form is *unique* and *invariant* for a fixed "form", and different "forms" of simplest normal forms are equivalent in the sense of simplicity. The explicit formulas have been implemented using the computer algebra system Maple, and an example is given to demonstrate the efficiency of the computer software.
Keywords. Differential equation, Hopf-zero bifurcation, conventional normal form, simplest normal form, symbolic computation, Maple.
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1 Introduction

Normal form theory plays an important role in the study of differential equations related to complex behavior patterns such as bifurcation and instability [5, 9, 10, 13]. It provides a convenient tool to compute a simple form of the original differential equations. The basic idea of normal form theory is employing successive, near identity nonlinear transformations to eliminate the so-called non-resonant nonlinear terms, and the terms called resonant which cannot be eliminated are remained in normal forms. Many research results related to this area may be found, for example, in the references [4, 5, 8].

It is well known that in general normal forms are not uniquely defined. For example, consider a codimension two example governed by the following differential equations: