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STRUCTURAL STABILITY OF A LINEAR SYSTEM IN A SATURATED MODE

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Abstract. We consider the ordinary differential equation

$$x' \in Tx + c - \partial I_D x,$$

used to describe a neural network. Here x takes values in $D = [-1, 1]^n$, T is linear, $c \in \mathbf{R}^n$ and the last term gives the outer normal cone at x. We give a set of conditions which are necessary and sufficient for structural stability, if $n \leq 2$.

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1. Introduction.

Professor S. Abe's book [1] uses the equation

$$x'(t) \in Tx(t) + c - \partial I_D x(t). \tag{1}$$

to describe a Hopfield neural network. This equation had been introduced in 1989 by Li, Michel and Porod [4]. They referred to it as a linear system in a saturated mode, or a linear system on a closed hypercube. Here x takes values in $D = [-1, 1]^n$, T is linear, and $c \in \mathbb{R}^n$. Note that the outer normal cone at x, $\partial I_D x$, is defined by: $w \in \partial I_D x$ iff $w \in \mathbb{R}^n$, $x \in D$, and for all $y \in D, (w, x - y) \geq 0$. A solution is defined in [3] to be a Lipschitz continuous function with (1) holding a.e.(t). By Theorem 3.1 of Brezis [2], together with the notes of Pazy [6], for $x_0 \in D$ there is a unique Lipschitz $x : [0, \infty) \to D$ with $x(0) = x_0$ and (1) holding a.e.(t).

For $B : D \to \mathbb{R}^n$, and $x \in D$, we define $B_a x \in \mathbb{R}^n$ as follows. For each i let

$$(B_a x)_i = \begin{cases} (Bx)_i & \text{if } |x_i| < 1, \\ min((Bx)_i, 0) & \text{if } x_i = 1, \\ max((Bx)_i, 0) & \text{if } x_i = -1. \end{cases}$$

Writing $T + c$ for the map $x \mapsto Tx + c$, we may write (1) as

$$x'(t) = (T + c)_a x(t) \qquad \text{a.e.}(t). \tag{2}$$