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ON SEMIGROUPS OF NONLINEAR OPERATORS AND THE SOLUTION OF THE FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. A class of nonlinear autonomous functional differential equations of retarded type is studied by associating with it an evolution equation in the space of initial data, the space $\mathcal{C}\left([-r,0];R^N\right)$. Existence, uniqueness and regularity results are proved.
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1 Introduction

Consider the following nonlinear autonomous differential equation:

$$\dot{x}(t) = F(x_t); \qquad x_0 = \varphi \in \mathcal{C}\left([-r,0];R^N\right). \tag{1}$$

where $x : [-r,T] \to R^N$, $0 < r < \infty$ is the delay and $x_t \in \mathcal{C}\left([-r,0];R^N\right)$ is the history of x at time t defined pointwise by $x_t(\theta) = x(t+\theta)$, for all $\theta \in [-r,0]$.

$F : \mathcal{C}\left([-r,0];R^N\right) \to R^N$ satisfies the following hypotheses:

(H_1) F is continuous and sends every bounded set of $\mathcal{C}$ into a bounded set of R^N.

(H_2) For all $\varphi^1, \varphi^2 \in \mathcal{C}$, and all $i \in \{1, ..., N\}$ such that:
$\left|\varphi_i^1(0) - \varphi_i^2(0)\right| = \left\|\varphi^1 - \varphi^2\right\|_C$, we have

$$\left(\varphi_i^1(0) - \varphi_i^2(0)\right)\left(F_i\left(\varphi^1\right) - F_i(\varphi^2)\right) \le 0$$

What we will prove here is that under (H_1) and (H_2), equation (1) has one and only one solution defined for all $t \ge 0$; the map $t \to x_t$ determines a strongly continuous nonlinear semigroup $T(t)$ on $\mathcal{C}$,

$$T(t)\varphi = x_t.$$

Moreover, the Crandall-Liggett approximation theorem applies to the evolution equation associated to (1) as follows:
denote by A the operator defined by

$$A\varphi = \dot{\varphi}, \quad D(A) = \left\{\varphi \in \mathcal{C}^1\left([-r,0];R^N\right); \dot{\varphi}(0) = F(\varphi)\right\} \tag{2}$$