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# ON THE NUMBER OF LIMIT CYCLES FOR SOME PERTURBED HAMILTONIAN POLYNOMIAL SYSTEMS

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**Abstract.** In this paper, we consider the perturbations of two Hamiltonian centers with Hamiltonians

$$H(x,y) = \frac{1}{2n}x^{2n} + \frac{1}{2m}y^{2m}, \quad H(x,y) = \frac{1}{2}y^2 + \frac{1}{2}x^2 + \frac{1}{2m}x^{2m},$$

respectively. For the former, we give the greatest number of isolated zeros (taking into account their multiplicity) of a class of Abelian integrals related to the corresponding perturbed Hamiltonian systems, and consequently obtain the indicated number of limit cycles from the perturbations of the corresponding Hamiltonian center in the class of differential polynomial systems. For the latter, we give the relative cohomology decomposition of the corresponding polynomial one form, and so obtain an estimate number of isolated zeros of the corresponding Abelian integral. We also study the maximum number of limit cycles that the perturbed systems can have surrounding a singular point.
**Keywords.** center, Hamiltonian system, Abelian integal, limit cycle, bifurcation.
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## 1 Introduction

By definition a *polynomial system* is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = R(x,\, y), \qquad \frac{dy}{dt} = \dot{y} = S(x,\, y), \tag{1}$$

where the dependent variables $x$ and $y$, and the independent one (the *time*) $t$ are all real, and $R, S \in \mathbf{R}[x,\, y]$, as usual $\mathbf{R}[x,\, y]$ denotes the ring of polynomials in the variables $x$ and $y$ with real coefficients. We say that $m = \max\{\deg R, \deg S\}$ is the *degree* of the polynomial system.

In the qualitative theory of planar real polynomial systems, the main open problem is the determination of the number and distribution of the limit cycles, that is, the second part of the Hilbert's sixteenth problem in [6]. One of the methods to obtain limit cycles is to perturb the periodic orbits of a center. Let $H(x,y)$ be a real polynomial of degree $m$, and let $P(x,y)$ and $Q(x,y)$ be real polynomials of degree $n$. Assume that $H(x,y) = h$ represent