

STRONG APPROXIMATIONS OF STOCHASTIC INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. We propose an Euler-type stochastic difference scheme for systems of stochastic integro-differential equations under Hölder type continuity assumptions for system coefficients. We estimate uniform p -th order rate of convergence by means of Gronwall and maximal submartingale inequalities. We also give examples of numerical simulation by means of the presented approximation method.

Keywords. Stochastic integro-differential equations, Euler difference scheme, polygonal interpolation, error estimation, Gronwall inequality.

AMS (MOS) subject classification: 65C20, 65C30

1 Introduction

We consider approximation of solutions to stochastic integro-differential equations of the following form:

$$dX_t = f(t, X_t, \int_0^t \alpha(t, s, X_s) ds) dt + \sigma(t, X_t, \int_0^t \beta(t, s, X_s) ds) dW_t \quad (1)$$

with the initial condition: $X_0 = x_0 \in R^d$.

Such equations arise in the study of stochastic systems in the presence of hereditary influences on the state variable of. Examples of such equations can be found in mathematical modelling of systems with hysteresis [?, ?]. The presence of a stochastic component makes the evolution of the state variable a stochastic process and accounts for intrinsic or external random influences. In [?] we considered the problem of convergence of a specialized Euler-Maruyama type stochastic difference scheme for (1.1) under some mild continuity assumptions for the coefficients. In the present paper we are concerned with convergence of similar approximation schemes under Lipschitz/Hölder continuity and sublinearity conditions for the system coefficients. As a result of the stronger regularity assumption we can estimate the rate of convergence of the approximate solution. We prove our results for the p -th order mean type convergence. The main theorem is an extension of