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DIRECTIONAL CONTROL OF THE MOTION OF A ROLLING DISK BY USING AN OVERHEAD ROTOR

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Abstract. This work deals with the guidance and control of a system which is composed of a disk rolling on the horizontal (X, Y)-plane, a controlled slender rod that is pivoted through its center of mass about the disk's center, and a rotor with its axis fixed in the upper end of the rod (see Fig. 1). The rod is controlled in such a manner that it is always aligned along the line passing through the points $\mathbf{O}$ and $\mathbf{C}$,where $\mathbf{O}$ denotes the center of the disk and $\mathbf{C}$ denotes the point of contact between the plane and the disk.The rotor rotates in a plane $\mathcal{T}$ that is always perpendicular to the rod.Furtheremore,when the plane of the disk is vertical to the (X, Y)-plane,then, $\mathcal{T}$ is parallel to the (X, Y)-plane. Given N points P_i , $i = 0, ..., N-1$ in the horizontal plane, N real numbers ϕ_{if} , $i = 0, ..., N-1$,a finite time interval $[0, t_f]$, and a sequence of times $\tau_0 = 0 < \tau_1 < ... < \tau_{N-1} = t_f$. Denote by ϕ the direction of the disk. Based on a dynamical model of the system,and by using the concept of path controllability,control laws are derived for the disk's directional,inclination and pedalling moments such that $[\mathbf{O}, \phi]$ will pass through $[P_j, \phi_{jf}]$ at the time $\tau_j, j = 0, ..., N-1$,respectively.

Key Words: Rolling disk, controlled pivoted slender rod, overhead rotor, directional control , nonholonomic constraints, path controllability.

1 Introduction

This work deals with the guidance and control of a system which is composed of a disk rolling on the horizontal (X, Y)-plane, a controlled slender rod that is pivoted through its center of mass about the disk's center, and a rotor with its axis fixed in the upper end of the rod (see Fig. 1). The rod is controlled in such a manner that it is always aligned along the line passing through the points $\mathbf{O}$ and $\mathbf{C}$,where $\mathbf{O}$ denotes the center of the disk and $\mathbf{C}$ denotes the point of contact between the (X, Y)-plane and the disk.The rotor rotates in a plane $\mathcal{T}$ that is always perpendicular to the rod.Furtheremore,when the plane of the disk is vertical to the (X, Y)-plane,then, $\mathcal{T}$ is parallel to the (X, Y)-plane.
In this work,first it is shown that the torque to the above-mentioned overhead rotor induces a directional moment to the disk's motion.Also,it is assumed here that two other moments,that is,a tilting moment and a pedalling moment are actuating the motion of the disk. Second,the following