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OPTIMAL STRATEGIES FOR VEHICLE CONTROL PROBLEMS WITH FINITE CONTROL SETS

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Abstract. We consider autonomous optimal control problems with finite controls sets and with time constraints. A typical example is driving a vehicle along a prescribed path within a given time while employing only finitely many control settings and attempting to minimize the fuel consumption. We reformulate the problem by eliminating the time variable. The resulting problem has the surprising feature that for certain intervals the Euler-Lagrange equation is an algebraic equation, rather than a differential equation. This provides useful information on the structure of the optimal controls which are composed of segments with pure control and segments with chattering control. We study the switching points between these two control types, and the order in which pure controls may appear in an optimal trajectory.

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1. Introduction

In this paper we consider a control problem with discrete control. The problem can be understood in practical terminology as a generalised vehicle control problem. We wish to drive a vehicle from one point to another within a given time in such a way that fuel consumption is minimised. We assume that only certain discrete control settings are possible and that each setting determines a constant rate of fuel supply. This is a typical situation with long haul freight trains. For any fixed sequence of controls the minimum fuel consumption is determined by selecting the optimal switching times. See for example Cheng and Howlett [3,4], Howlett *et al* [8,9], Howlett and Pudney [10], Howlett [11], Howlett and Cheng [12] and Pudney and Howlett [16]. But here we will take a more general view. We seek an optimal strategy with no restriction on the sequence of controls and no limit on the number of switching points. Indeed we will show that in normal circumstances the optimal strategy contains some idealized segments of chattering control which must be approximated in practice by a sequence of pure controls with a large number of switches.

Our considerations of this problem were inspired by a study of energy-efficient driving strategies for long-haul freight trains. Early work on train control problems by Asnis *et al* [1] and Howlett [6,7] assumed continuous levels of control and used applied acceleration as the control variable. Later