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THE PARABOLIC CAUCHY PROBLEM AND QUENCHING

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Abstract. This article treats the Cauchy problem for the equation

$$u_t = \Delta u + f(t, x, u)$$

with initial values $u(0, x) = \varphi(x)$ in $\mathbb{R}^N$ with an eye to the study of quenching phenomena. Among the results are an existence theorem under the sole assumption of continuity of f, existence of maximal and minimal solutions without a monotonicity assumption regarding f, an extension of the results on growth of solutions and on uniqueness that were obtained by Aguirre, Escobedo, and Herrero in [1] and [3] for power functions $f(u) = u^p$ with $0 < p < 1$ to a larger class of nonlinearities, using a new technique, and furthermore, new results on the behavior at infinity of the solution in just one fixed direction when the behavior of the initial function in this direction is known. The results are extended to parabolic systems, and some applications to quenching problems are discussed.
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1. Introduction

It is the purpose of this article to introduce and study the quenching phenomenon for the parabolic initial value problem where the initial values are prescribed in $\mathbb{R}^N$. Let us consider for illustration the by now classical equation introduced by Kawarada in [6], which was treated by sub- and supersolution techniques in [8]

$$u_t \;=\; \Delta u + \frac{1}{1-u} \;\text{ in } [0,T) \times \mathbb{R}^N, \quad u(0,x) \;=\; \varphi(x) < 1 \;\text{ on } \mathbb{R}^N.$$

The basic questions are familiar from the initial-boundary value problems: When does quenching occur and what is the maximum value of T, where quenching occurs? The answer depends on φ. What is the set in x-space where the solution approaches 1 as time t tends to T, and how does the solution behave near T? The lack of boundary conditions raises a new question: How does the solution behave at infinity?

If the initial value φ is a constant $\alpha < 1$, quenching is an ODE problem which is trivial and of no interest in itself but may serve as a means of