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QUENCHING BEHAVIOR FOR DEGENERATE PARABOLIC PROBLEMS

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Abstract. This article studies the degenerate parabolic differential equation, $u_{xx} - x^q u_t = -f(u)$, where q is any real number and $f \in C^2[0, c)$ for some constant c such that $f(0) > 0$, $f' > 0$, $f'' \geq 0$ and $\lim_{u \to c^-} f(u) = \infty$. With nonnegative initial data and zero boundary condition, location of quenching and conditions for single-point quenching are discussed. An upper bound for quenching time for single-point quenching is also established.

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1. Introduction

Let $T \leq \infty$, $a > 0$, $\Omega = (0, a) \times (0, T)$, $\partial\Omega$ be the parabolic boundary $([0, a] \times \{0\}) \cup (\{0, a\} \times (0, T))$ of Ω, and

$$Lu \equiv u_{xx} - x^q u_t$$

where q is any real number. When $q \neq 0$, the differential operator is said to be degenerate. Floater [5] studied the degenerate parabolic equation $Lu = -u^p$ in Ω, for $p > 1$ and $a = 1$, subject to certain nonnegative initial data and zero boundary conditions. He showed existence of a unique classical solution and proved that for $p \leq q + 1$, the solution blows up at $x = 0$ in finite time. The case $p > q + 1$ was studied by Chan and Liu [4]. In particular, they established that the blow-up set is a proper compact subset of $(0, a)$, and that if the initial data are sufficiently small, the global soluton is uniformly bounded above. Recently, Chan and Kong [1, 3] studied the degenerate quenching problem,

$$Lu = -f(u) \text{ in } \Omega, \; u = 0 \text{ on } \partial\Omega \tag{1}$$

for $q \neq 0$ with $f \in C^2([0, c))$ for some positive constant c such that $f(0) > 0$, $f' > 0$, $f'' \geq 0$ and $\lim_{u \to c^-} f(u) = \infty$. By quenching phenomena, we mean the blow-up of u_t in finite time T (called the quenching time) and existence of a unique critical length a^* (which is the length such that for $a < a^*$, u exists for all $t > 0$, and for $a > a^*$, u reaches c somewhere in finite time and u_t blows up there). When $u_t > 0$, a necessary condition for quenching to occur is

$$\max\{u(x, t) : 0 \leq x \leq a\} \to c^- \text{ as } t \to T^-.$$