Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 53-68

ON THE STEADY STATES OF A NONLOCAL SEMILINEAR HEAT EQUATION

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Abstract. The purpose of this paper is to study the steady state solutions for a nonlocal semilinear heat equation. The nonlocal term involves the integral of the solution over the spatial interval. By using some elementary differential inequalities, a detailed analysis of existence and/or nonexistence of steady states are given for different ranges of parameters, from which it follows that some global existence and nonexistence results of solutions of the original heat equation can be easily deduced.

AMS (MOS) subject classification: 34B15, 35K20

1. Introduction

In this paper, we study the steady states for the following nonlocal semilinear heat equation

$$u_t = u_{xx} + \varepsilon \|u(\cdot,t)\|^q/(1-u)^\beta, 0 < x < 1, t > 0, \tag{1.1}$$

with the zero Dirichlet boundary condition and with the initial condition

$$u(x,0) = u_0(x), 0 < x < 1,$$

where $\varepsilon > 0$, $q > 0$, $\beta > 1$, $0 \le u_0(x) < 1$, $x \in [0,1]$, and $\|\cdot\|$ is the L^1-norm defined by

$$\| u(\cdot,t) \| = \int_0^1 |u(x,t)| dx.$$

Problems like the above one for parabolic equations involving nonlocal terms have drawn a lot of attentions during past years. We list some (but, far from complete) recent articles related to nonlocal term involving the integral of the solution over the spatial domain in the references (see, e.g., [1-7, 9-12]).

Notice that by the standard theory for parabolic boundary value problem there exists a unique classical solution for a small time interval (cf. [3]). The solution u of (1.1) is said to be quenching if the maximum of $u(\cdot,t)$ reaches 1 in some finite time. In this case the equation becomes singular.

By using the information of steady states, it is well-known that some quenching criteria, i.e., global (in time) existence and nonexistence results,