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INITIAL DATA FOR A SINGLE-POINT QUENCHING

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Abstract. This article studies the following parabolic initial-boundary value problem,

$$u_t - u_{xx} = -\epsilon u^{-p} \text{ in } (-1,1)\times(0,T),$$

$$u(x,0) = u_0(x) \text{ for } -1 \le x \le 1,\ u(-1,t) = 1 = u(1,t) \text{ for } 0 < t < T,$$

where p and ϵ are positive constants, $0 < T \le \infty$, $0 < u_0(x) \le 1$, $u_0(-1) = 1 = u_0(1)$, and u_0 is symmetric with respect to the line $x = 0$. Criteria on the initial data for a single-point quenching are established. To illustrate the main results, two examples are given.
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1. Introduction

Let $0 < T \le \infty$, p and ϵ be positive constants, $Hu = u_t - u_{xx}$, $D = (-1,1)$, $\bar{D} = [-1,1]$, and $\Omega = D \times (0,T)$. We consider the first initial-boundary value problem:

$$\left.\begin{array}{c} Hu = -\epsilon u^{-p} \text{ in } \Omega, \\ 0 < u(x,0) = u_0(x) \le 1 \text{ on } \bar{D},\ u(\pm 1,t) = 1 \text{ for } 0 < t < T, \end{array}\right\} \qquad (1.1)$$

where $u_0(-1) = 1 = u_0(1)$. A solution u is said to quench if there exists a finite time T such that

$$\min\{u(x,t) : x \in \bar{D}\} \to 0^+ \text{ as } t \to T^-.$$

The point $x^* \in \bar{D}$ is a quenching point of the problem (1.1) if $\lim_{t\to T^-} u(x^*,t) = 0$. It follows from Chan and Ke [2], and Chan and Yang [3] that for $u_0(x) \equiv 1$, there exists a value $\epsilon^* > 0$ such that the minimum of the solution u reaches zero in finite time if $\epsilon > \epsilon^*$ while the solution exists globally and is bounded away from zero if $\epsilon \le \epsilon^*$. Since a solution with positive initial data u_0, satisfying

$$\text{the maximal steady-state solution of the problem (1.1)} \le u_0 \le 1,$$