Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 121-128

DOES QUENCHING FOR DEGENERATE PARABOLIC EQUATIONS OCCUR AT THE BOUNDARIES?

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Abstract. Let q be any nonnegative real number. This article studies the degenerate equation,

$$x^q u_t - u_{xx} = f(u) \text{ in } (0, a) \times (0, T),$$

subject to the initial condition $u(x, 0) = 0$ for $0 \leq x \leq a$, and respectively, second, and third boundary conditions. Here, $f > 0$, $f' > 0$, $f'' \geq 0$, and $\lim_{u \to c^-} f(u) = \infty$ for some positive constant c. It is shown that for second boundary conditions, quenching occurs at $x = 0$ for $q > 0$ while for third boundary conditions, $x = 0$ and $x = a$ are not quenching points.

AMS (MOS) subject classification: 35K65, 35K57, 35K60, 35K20

1. Introduction

Let a be a positive constant, $T \leq \infty$, $D = (0, a)$, $\bar{D} = [0, a]$, $\Omega = D \times (0, T)$, $\Gamma_1 = \{0\} \times (0, T)$, $\Gamma_2 = \{a\} \times (0, T)$, and $Lu = x^q u_t - u_{xx}$, where q is any nonnegative real number. Let us consider the following initial-boundary value problem:

$$Lu = f(u) \text{ in } \Omega, \tag{1.1}$$

$$u(x, 0) \equiv 0 \text{ on } \bar{D},\ u_x + \alpha_i u = 0 \text{ on } \Gamma_i,\ i = 1, 2, \tag{1.2}$$

where α_i ($i = 1,\ 2$) are constants such that $\alpha_i \equiv 0$ for second boundary conditions, and $\alpha_1 < 0$ and $\alpha_2 > 0$ for third boundary conditions. Here, $f > 0$, $f' > 0$, $f'' \geq 0$, and $\lim_{u \to c^-} f(u) = \infty$ for some positive constant c. The solution u is said to quench if $\lim_{t \to T^-} \max_{0 \leq x \leq a} u(x, t) = c$. Recently, Chan and Kong [2] studied (1.1) with first homogenous boundary conditions. They showed that there exists $a^* > 0$, such that for $a < a^*$, u exists for all