Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **8 (2001)** 1-14

DAMAGE MODELS OF ELASTIC MATERIALS

C. Y. Chan[1] and L. Ke[2]

Department of Mathematics
University of Louisiana at Lafayette
Lafayette, LA 70504-1010, U.S.A.
[1] e-mail: chan@louisiana.edu
[2] e-mail: ke@louisiana.edu

Abstract. For a rod of elastic material, it is shown that the external force must be greater than a certain threshold value in order for the rod to get damaged. For a constant external force, there exists a critical length a^* beyond which the rod breaks; if the rod is broken, it breaks abruptly, that is, the time derivative of damage is infinite when the rod breaks.
AMS (MOS) subject classification: 35K20,35K57, 35K60, 73B99, 73C99

1. Introduction

Damage of materials has been modelled for a long time by damage quantities within the framework of continuum mechanics. They are internal quantities appearing in the expression of the free energy of the material. The many possible expressions for the free energy yield numerous and versatile constitutive laws. The constitutive laws are coupled with the balance law obtained from the principle of virtual power to give a predictive theory. Damage, such as damage of concrete, results from microscopic movements. Recently, Frémond and Nedjar [4] modified the power of the interior forces to depend not only on the strain rates, but also on the damage rate and the gradient of the damage rate. The damage rate is related to microscopic movements while the gradient of the damage rate accounts for the influence of damage at a material point on the damage of its neighborhood. Based on their work, we obtain here a one-dimensional mathematical model for the evolution of the damage.

Let us consider a one-dimensional rod of length a. It is fixed at its left end while a traction p acts on its right end. Let $v(\tilde{x}, \tilde{t})$ be the displacement and $u(\tilde{x}, \tilde{t})$ be the damage. When $u = 1$, the material is undamaged; when $u = 0$, it is completely damaged, that is, the rod is broken. When $0 < u < 1$, the material has some open cracks, but it retains its elastic behavior (with modified (softer) elastic moduli).